<<守恒定律用的数值法>>

图书基本信息

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内容概要

these notes developed from a course on the numerical solution of conservation laws first taught at the university of washington in the fall of 1988 and then at eth during the following spring. the overall emphasis is on studying the mathematical tools that are essential in developing, analyzing, and successfully using numerical methods for nonlinear systems of conservation laws, particularly for problems involving shock waves. a reasonable understanding of the mathematical structure of these equations and their solutions is first required, and part i of these notes deals with this theory. part ii deals more directly with numerical methods, again with the emphasis on general tools that are of broad use. i have stressed the underlying ideas used in various classes of methods rather than presenting the most sophisticated methods in great detail. my aim was to provide a sufficient background that students could then approach the current research literature with the necessary tools and understanding.

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章节摘录

Discontinuous solutions of the type shown above clearly do not satisfy the PDE in theclassical sense at all points， since the derivatives are not defined at discontinuities. Weneed to define what we mean by a solution to the conservation law in this case. To findthe correct approach we must first understand the derivation of conservation laws fromphysical principles. We wilI see in Chapter 2 that this leads first to an integral form of theconservation law， and that the differential equation is derived from this only by imposingadditional smoothness assumptions on the solution. The crucial fact is that the integralform continues to be valid even for discontinuous solutions. Unfortunately the integral form is more difficult to work with than the differentialequation， especially when it comes to discretization. Since the PDE continues to holdexcept at discontinuities， another approach is to supplement the differential equations byadditional “ jump conditions ” that must be satisfied across discontinuities. These can bederived by again appealing to the integral form. To avoid the necessity of explicitly imposing these conditions， we will also introducethe weak form of the differential equations. This again involves integrals and allowsdiscontinuous solutions but is easier to work with than the original integral form of theconservation laws. The weak form will be fundamental in the development and analysisof numerical methods.

Another mathematical difficulty that we must face is the possible nonuniqueness ofsolutions. Often there is more than one weak solution to the conservation law with thesame initial data. If our conservation law is to model the real world then clearly onlyone of these is physically relevant. The fact that the equations have other， spurious， solutions is a result of the fact that our equations are only a model of reality and somephysical effects have been ignored. In particular， hyperbolic conservation laws do notinclude diffusive or viscous effects. Recall， for example， that the Euler equations resultfrom the Navier-Stokes equations by ignoring fluid viscosity. Although viscous effectsmay be negligible throughout most of the flow， near discontinuities the effect is alwaysstrong. In fact， the full Navier-Stokes equations have smooth solutions for the simpleflows we are considering， and the apparent discontinuities are in reality thin regions withvery steep gradients. What we hope to model with the Euler equations is the limit ofthis smooth solution as the viscosity parameter approaches zero， which will in fact be oneweak solution of the Euler equations.

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